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- February 27: "Who's who in modern mathematics" by Professor Ernest B. Skinner.
- March 13: "The trisection of an angle by means of conic sections" by Madge Ryan '20; "Curves" by Ethel Vasey '19.
- April 16: Social evening at the home of Professor and Mrs. Dowling. Illustrated lecture on "Famous mathematicians" by Professor Dowling.
- May 8: "Works of Archimedes" by Professor Charles S. Slichter.
- May 22: Business meeting for election of officers. Officers elected for the year 1919-20: President, Margaret Lee '20; vice-president, Ruth-Marie Urban '20; secretary-treasurer, Gladys Baur '20.
- October 21: Picnic.
- October 30: "Classification of curves and surfaces" by Professor Van Vleck.
- November 13: "Some surprises in the history of mathematics" by Professor Dowling.
- December 4: "Mathematical recreations" by Estelle Stone '20 and Madge Ryan '20.
- December 18: "Graphical calculation" by Professor Arnold Dresden.
- January 8, 1920: "Numerals of antiquity" by Ruth-Marie Urban '20; "Life and works of Abel" by Alta Gudsos '20.
- January 22: "Integers" by Professor Skinner.

PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND OTTO DUNKEL.

Send all communications about problems and solutions to **B. F. FINKEL**, Springfield, Mo.

NOTE ON CAUSTICS.

By OTTO DUNKEL, Washington University.

The purpose of this note is to show that the formula (3), which was derived by Professor da Cunha in another way in his solution of problem 2768, below, has a somewhat wider application. If from a point F rays of light fall upon the concave side of a curve Γ in a plane containing the point, and the reflected rays intersect on the same side, the envelope γ of these reflected rays is called the caustic of Γ with respect to the point F . Let M be a point of Γ , F' the point in which the ray reflected from M has contact with γ , ω the angle of incidence, $R = MO$, the radius of curvature of Γ at M , $\delta = FM$ and $\delta' = MF'$. Then it may be proved that

$$(1) \quad \frac{1}{\delta} + \frac{1}{\delta'} = \frac{2}{R \cos \omega}.$$

A simple geometrical derivation of this result is given in Humbert's *Cours d'Analyse*, vol. 1, page 77. Let the point of intersection of the normal to Γ at M with the line FF' be denoted by T and the length MT by n . By dropping

perpendiculars from F and F' upon the normal MT and by considering two pairs of similar triangles, it is easily seen from a figure that

$$(2) \quad \frac{n - \delta \cos \omega}{\delta' \cos \omega - n} = \frac{\delta}{\delta'} \quad \text{or} \quad \frac{1}{\delta} + \frac{1}{\delta'} = \frac{2 \cos \omega}{n}.$$

From (1) and (2) there results at once

$$(3) \quad R = \frac{n}{\cos^2 \omega},$$

the formula mentioned above.

If the curvature of the curve Γ is such that the point F' lies on the side opposite to F , then by a very slight modification of the derivation given by Humbert it may be shown that (1) is again true provided that δ' be regarded as negative. In this case there is no caustic by reflection in the physical sense and it is not considered by Humbert. Also a proof similar to that indicated above shows that (2) also is true if δ' be regarded as negative. Thus (3) is true for both cases. It is thus somewhat more general and at the same time simpler than (1); it also has the advantage of being more convenient for geometrical constructions. Thus, if the point O has been found on the evolute of a curve Γ corresponding to the point M of Γ , the formula (3) gives an easy construction for the point F' on the caustic of Γ with respect to a given point F . If, for example, the curve Γ is a circle the caustic may be easily traced by this construction. If, on the other hand, Γ is a conic and F is a focus, then γ reduces to the other focus, or to a point at infinity in the case of a parabola, and the formula gives the construction given by Professor da Cunha.

Another construction for the center of curvature may be derived from a property of triangles. In any triangle FMF' let C be the middle point of the side FF' , T the point in which the bisector of the angle at M meets FF' , P the point in which the side FM is cut by the perpendicular to the bisector at T , O the point in which the perpendicular to FM at P meets the bisector MT (O is the center of curvature when F , F' and M have the meaning above) and finally Q the point in which TP meets CM . The property referred to is the fact that QO is perpendicular to FF' . A similar theorem is true for the external bisector. The second construction, which applies to the general case as well as to conics, is then as follows: Erect a perpendicular to the normal at the point where it cuts the axis and from the point in which this perpendicular meets the line joining the center with the given point on the curve drop a perpendicular to the axis and produce it to meet the normal. This point is the center of curvature.

PROBLEMS FOR SOLUTION.

2829. Proposed by E. S. PALMER, New Haven, Conn.

Given a set of arbitrary pairs of positive integers (a_p, b_p) , $(p = 1, 2, \dots, n)$: (a). Is it always possible to find a set of positive integers k_p , $(p = 1, 2, \dots, n)$ such that

$$k_p a_p + k_p b_p > \sum_{r=1}^{r=n} k_r a_r, \quad (p = 1, 2, 3, \dots, n).$$

(b) If or when possible, show how to find k_p ?